Mathematical Analysis - List 12

1. If f is a continuous function such that $\int\limits_0^x f(t)dt = xe^{2x} - \int\limits_0^x e^{-t}f(t)dt$ for all x, find an explicit formula for $f(x)$.

2. Evaluate $\lim\limits_{x\to 0} \frac{1}{x}\int\limits_0^x (1-\operatorname{tg} 2t)^{1/t}dt$.

3. Suppose that the graph of a differentiable function f passes through the origin and the point $(1,1)$. Find the value of the integral $\int\limits_0^1 f'(x)\,dx$.

4. If f' is continuous on $[a,b]$, show that $2\int\limits_a^b f(x)f'(x)dx = [f(b)]^2 - [f(a)]^2$.

5. Let f'' and g'' be continuous and $f(0)=g(0)=0$. Show that

$$\int\limits_0^a f(x)g''(x)dx = f(a)g'(a) - f'(a)g(a) + \int\limits_0^a f''(x)g(x)dx.$$

6. Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y and find the area of the region.

a) $y=1/x,\ \ y=1/x^2,\ \ x=1,\ \ x=2$;

b) $y=x^2,\ \ y^2=x$;

c) $y=e^x,\ \ y=e^{3x},\ \ x=1$;

d) $y=x^4-x^2,\ \ y=1-x^2$;

e) $x=1-y^2,\ \ x=y^2-1$;

f) $x+y^2=2,\ \ x+y=0$.

7. Sketch the region that lies between the curves $y=\cos x$ and $y=\sin 2x$ and between $x=0$ and $x=\pi/2$. Notice that the region consists of two separate parts. Find the area of this region.

8. Find the area of the region bounded by the parabola $y=x^2$, the tangent line to this parabola at $(1,1)$, and the x-axis.

9. Find the volume of the described solid S:

a) The base of S is a circular disk with radius r. Parallel cross-sections perpendicular to the base are squares.

b) The base of S is the region $\{(x,y) : x^2 \leqslant y \leqslant 1\}$. Cross-sections perpendicular to the y-axis are equilateral triangles.

10. Find the volume of the solid obtained by rotating the region bounded by the curves about the specified axis.

a) $y=e^x$, $y=0$, $x=0$, $x=1$, about the x-axis;

b) $y=x^2$, $y^2=x$, about the x-axis;

c) $y=e^x$, $y=1$, $x=1$, about the y-axis;

d) $y=x^4$, $y=1$; about $y=2$.